

Determination of the
Temperature Coefficient, and
Coefficient of Resistivity
Of Copper, Iron and Aluminum

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Determination of the
temperature coefficient,

DETERMINATION OF THE TEMPERATURE
COEFFICIENT, AND COEFFICIENT OF RESISTIVITY
OF COPPER, IRON AND ALUMINUM

A THESIS

PRESENTED BY

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ARMOUR INSTITUTE OF TECHNOLOGY

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1.

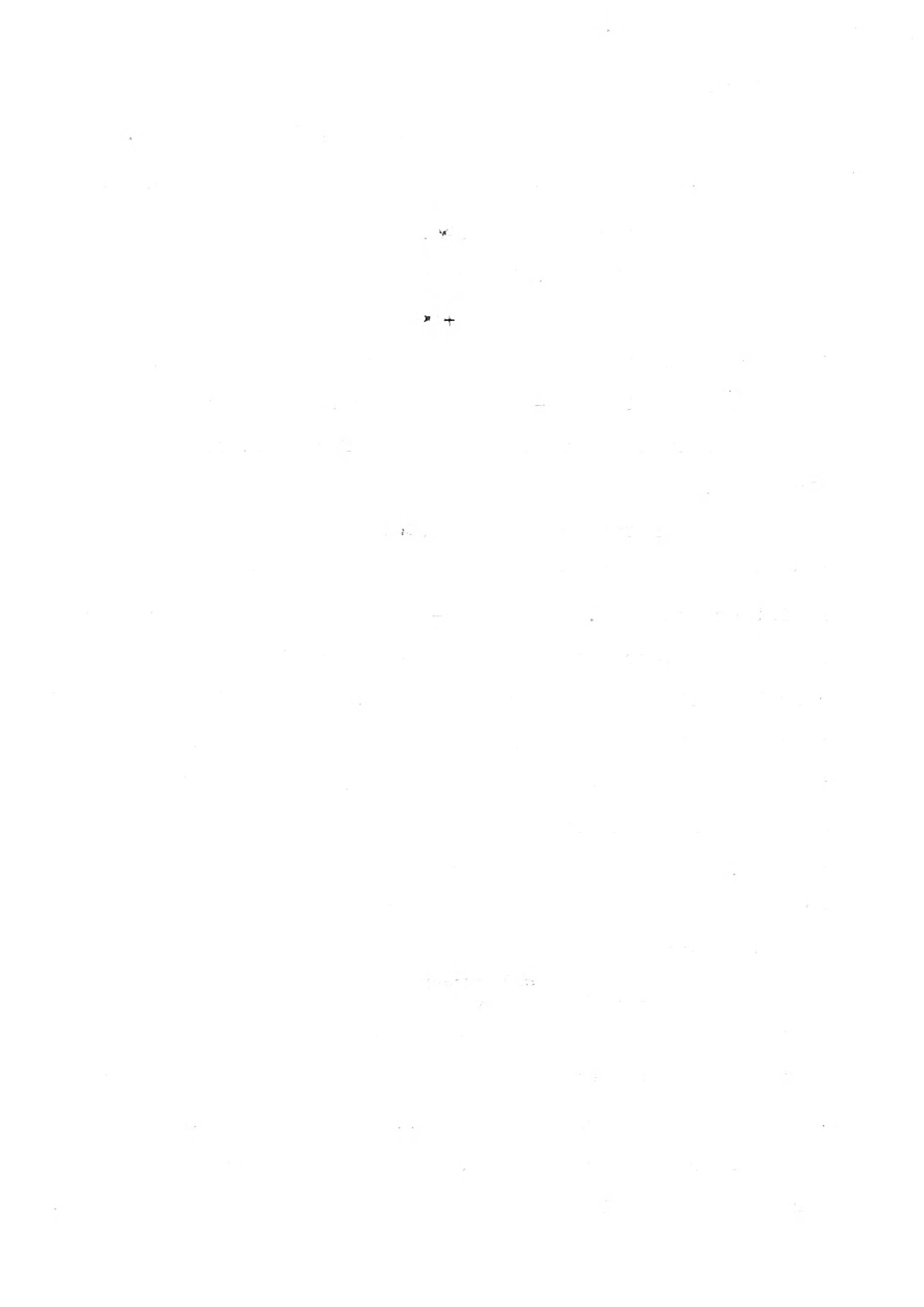
DETERMINATION OF TEMPERATURE COEFFICIENT AND COEFFICIENT
OF RESISTIVITY OF COPPER, IRON, AND ALUMINIUM.

The object of the following experiments was to find the mean temperature coefficient, α , between the temperatures of 0° and 100° Centigrade, in the formula

$$R_t = R_0(1 + \alpha t)$$

in which R_0 is the resistance of a conductor at 0° C. and R_t at t° C.; also the mil-foot resistance, and the volume and mass resistivity of various samples of copper, iron and aluminium.

The temperature coefficient, α , is defined to be the increase in resistance of a conductor per degree per ohm resistance at 0° C. The mil-foot resistance of a conductor is the resistance of a conductor one foot long and one circular mil in cross sectional area. The volume resistivity of any material is defined to be the resistance of a cube of the material one centimeter on a side. The usual manner of stating this resistivity is in michrons per centimeter cube at 0° C. To determine the volume resistivity of a sample of a metal or alloy, it is desirable to possess it in the form of a carefully drawn wire of uniform circular cross section. Owing to the difficulty of ^{determining} the diameter of very fine wires, it is found more desirable and convenient to determine and define the resistivity of metals and alloys by the resistance in ohms per meter gramme at 0° C., that is to say, by stating the ohmic resistance at 0° C. of a wire of circular cross section having a length of one meter and weighing one gramme.



This is known as the mass resistivity.

The samples tested were copper, steel and aluminium wires as follows:

Sample 1. #10 B.&S. gauge, annealed copper wire obtained from John A. Roebling's Sons Co.

Sample 2. #14 B. & S. gauge annealed copper wire.

Sample 4. #14 B. & S gauge annealed copper wire.

Sample 5. #14 B. & S. gauge annealed copper wire.

Sample 6. #14 B. & S. gauge annealed copper wire.

Sample 9. # 8 B. & S. gauge annealed copper wire, obtained from the Okanite Co.

Sample 10. # 9 B. & S. gauge annealed copper wire, obtained from the American Steel & Wire Co.

Sample 2. # 8 B.W.G. steel wire, American Steel & Wire Co's Extra B.B.

Sample 7. Roebling's #8 B.B.

Sample 8. Roebling's #8 Extra B.B.

The three samples of steel were obtained through the courtesy of the Ristine Co.

Samples I and II of annealed Aluminium about #10 B.W.C. from the American Aluminium Co.

The resistances at various temperatures ranging from 0° to 100° C. were measured by means of the Thompson double bridge. A scheme of the bridge and the connections for the determination of the resistances is shown on an accompanying blue print. The principle of the bridge method is that of balancing the drop across the unknown resistance against the drop across a

known resistance, so that all the resistances of lead wires are eliminated. The wire to be tested was cut into samples twenty-two inches in length and a sample was placed in the box containing the oil bath for heating and fastened and held straight by means of the clamps, AA, (see photograph) which are connected to the series posts of the box. The wire was then connected in series with the known resistance, a Hartmann and Braun .001 ohm standard manganin resistance, and also in series with an ammeter, a carbon rheostat, a storage cell and a switch as shown in the scheme. The rheostat was adjusted so that a current of 15 amperes flowed through the series circuit. The heating box is so arranged that the knife edges, exactly 50 centimeters apart, rest upon the wire to be tested, between the series posts, so that the drop across 50 centimeters of the wire is balanced against the drop across the standard resistance. These knives, BB, in the photograph, are connected by large copper leads to the pressure binding posts on the top of the box. These posts were connected to the posts marked, xx, on the bridge and also the pressure posts on the standard resistance were connected to the posts marked, NN, on the bridge. The galvanometer was also connected as shown in the scheme. The galvanometer used was a Leeds and Northrop, suspended coil type, mounted in a wall case on springs so that the jarring of the building would have a minimum effect in deflecting the coil. The deflection was read by means of a telescope and scale so as to obtain a maximum accuracy in the adjustment of the resistances

n

nd

on the bridge for a balance, the coil showing no deflection at this point.

To obtain temperatures above that of the room, the oil bath surrounding the wire was heated by an electric heating coil. Readings of resistance were taken about every five degree increments in temperature up to 100° C., the oil being constantly stirred both by hand and electric motor. At each temperature where the resistance was measured the heat was partly shut off, only enough being left on to keep the temperature constant, the amount of heating current left on being adjusted through a lamp rack. After the temperature had remained constant for a short time, the dials on the bridge were adjusted, the two 100 plugs, as shown in the photograph being out, until the galvanometer showed no deflection; the temperatures and resistances were then read simultaneously and recorded.

The temperatures below that of the room were obtained by running cold brine through a coil of tin piping in the oil bath. The arrangement for running the brine through the coil from a tank is shown in a photograph of the apparatus. On account of the high congealing point of the oil used for the high temperatures, kerosene was substituted for the low temperatures. In this way the temperature of the bath was reduced to 0° C., readings of resistance being taken as before.

The temperatures were read by means of two accurate thermometers, reading from 00° to 100° C., and graduated to tenths of a degree, placed in the oil through the top of the heating

box. The mean temperature of the wire for each reading was taken as the average between the two thermometer readings. This mean temperature was corrected for the mercury in the stem projecting above the oil in the box by means of the formula

$$T = t - .000143 n (t' - t) \quad (\text{Smithsonian Tables})$$

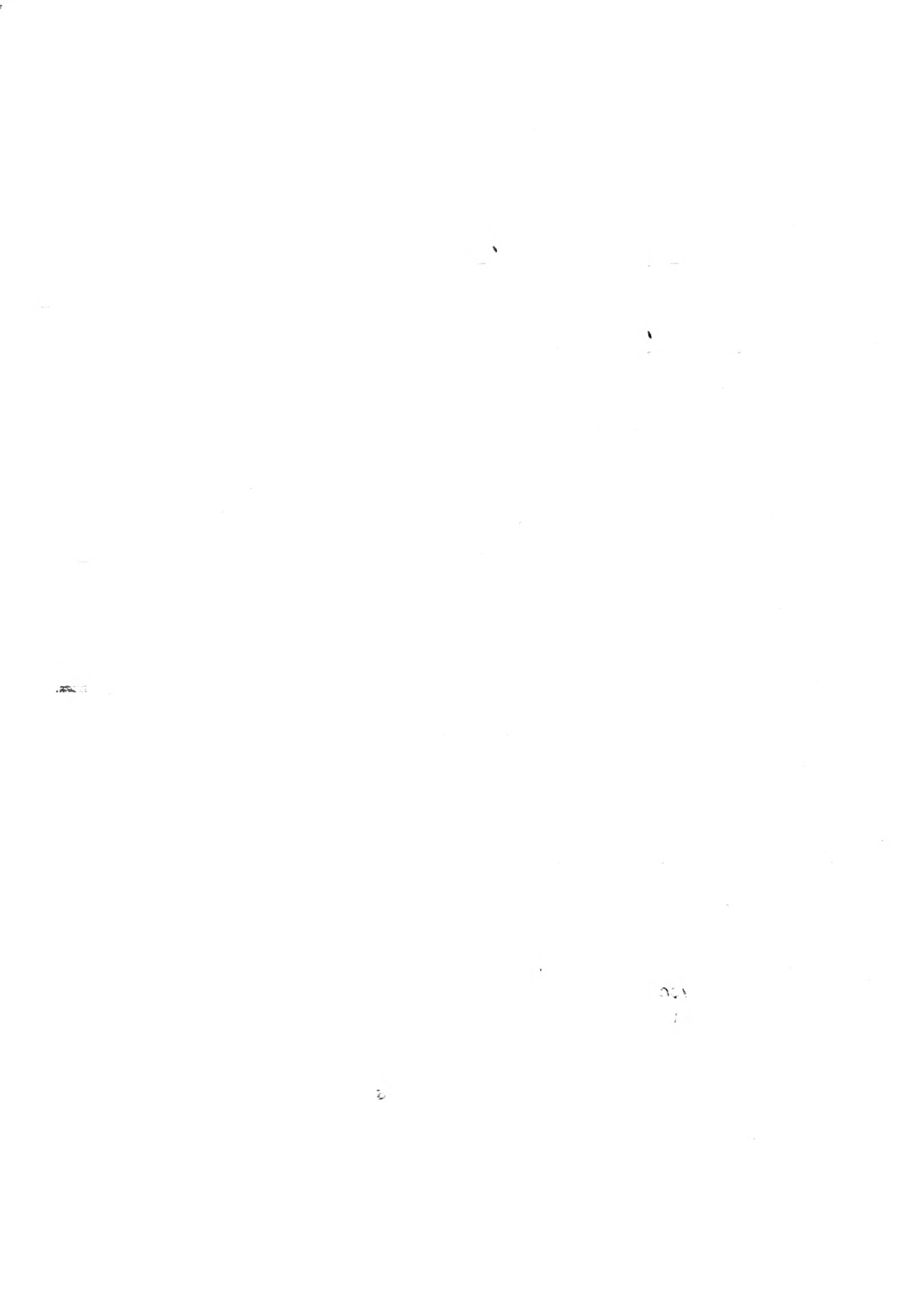
in which T is the corrected temperature, t , the observed temperature, and t' , the mean temperature of glass stem and mercury column; this temperature was assumed to be a little above room temperature; n is the length of the mercury column in the stem in scale degrees, this length being figured from the fifteen degree point, the mercury being subjected to the heat of the oil up to the fifteen degree mark. This correction amounts to about nine tenths of a degree at 100° C.

According to the certificate for the Hartmann and Braun standard resistance as tested by the Bureau of Standards, ~~its~~ its resistance at 20.4° C. is .00000246 international ohms. This value was used throughout as the resistance of the standard coil. If N is the reading on the dials of the bridge for a balance, then R , the resistance of fifty centimeters of the wire, is given by the equation

$$\frac{R}{.00000246} = \frac{N}{100} \quad \text{or} \quad R = N \times \frac{.00000246}{100}$$

when the two ¹⁰⁰plugs are out.

Curves were plotted as shown on sheets 1 to 10 with temperature as abscissae and resistances as ordinates. From these curves, which were found to be straight lines, the average temperature coefficient was obtained. The mean temperature coefficient was taken as the average coefficient figured from each of the points which fell on the curves.



The mil-foot resistance for each sample was found at 0° C. and at one or two other temperatures at which the resistance fell on the resistance-temperature curve. From these points straight line curves were drawn which show the relation of mil-foot resistance to temperature. In the same way straight line curves were drawn which show the relation of the mass and volume resistivity to temperature. The mil-foot temperature-resistance curves are shown on sheets 15 to 22 and those for the mass and volume resistivity on sheets 23 to 32.

The mass resistivity in ohms per meter gramme at 0° C. was found from the formula

$$\rho' = \frac{10^4 M R}{l^2}$$

in which ρ' = mass resistivity, M = mass ~~resistivity~~ of fifty centimeters of sample in grammes, R = resistance of sample in ohms at 0° C. and l = the length of the wire in centimeters.

The volume resistivity in microns per cubic centimeter at 0° C. was found from the formula

$$\rho = \frac{\rho' \times 10^2}{d}$$

where ρ is the volume resistivity at 0° C. and d is the density.

The density, d , of each sample was found by weighing it in and out of distilled water by means of an accurate chemical balance.

The diameters of the samples were found by means of micrometer calipers. Measurements were taken at several points along the wire, twice at each point, one reading being taken when the micrometer caliper was at right angles to the position when the first reading was taken. The diameter was



then taken as the average of all the readings.

In order to find out whether the samples of copper were hard drawn or annealed tests were made to find the tensile strength.

Data were secured as to the chemical composition and purity of all the copper samples except sample 1 as follows:

Sample	% Copper	% Tin
3	99.49	.5
4	99.	.47
5	99.18	.29
6	99.1	.52
9	98.41	.71
10	99.64	.28

In the above samples the tin appeared only as a coating which would not affect the value of the temperature coefficient or resistivity, the per cent being so small. Sample 1 had no such tin coating.

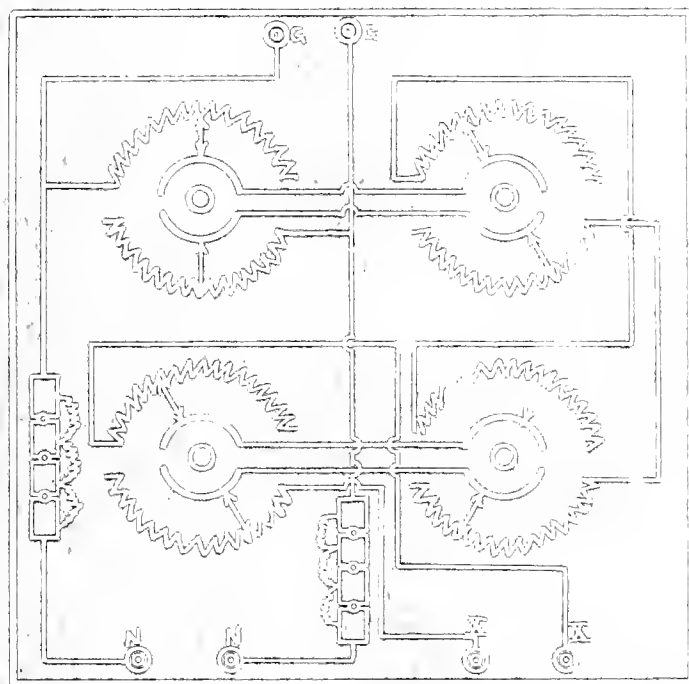
The average temperature coefficient for all the ^{copper} samples is .004221, ~~and~~ ^{value being} the maximum [^].004288 and the minimum, .004127. Any slight difference in the purity or treatment of copper will affect its temperature coefficient. The value for the temperature coefficient adopted in the A.I.E.E. Standardization Rules is .0042; the German rules adopt .004, while on the other hand the British Engineering Standards Committee use .00418. The value of the coefficient, .004221, corresponds very nearly to that of the A.I.E.E. Standardization Rules.

The variation of the temperature coefficient for the steel

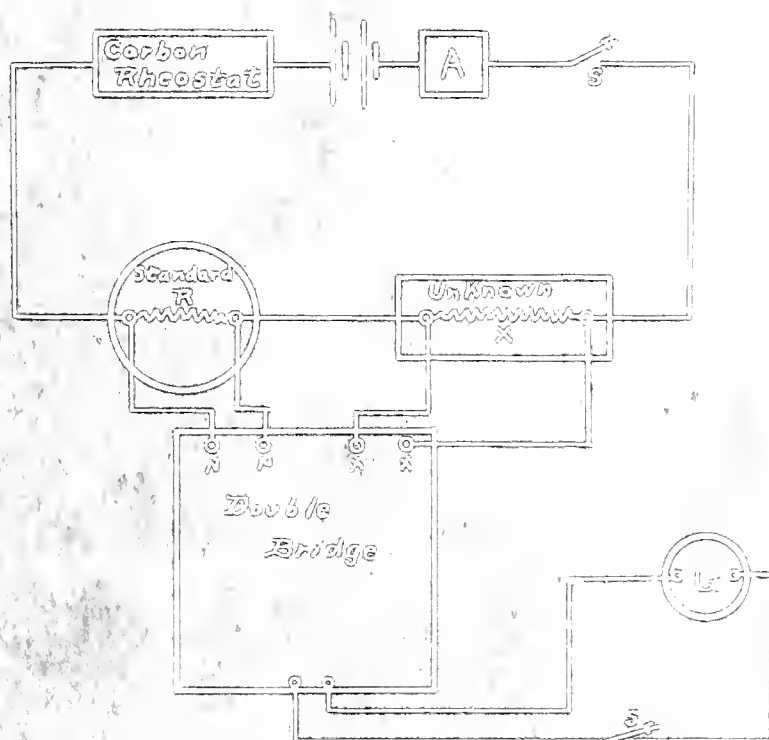
wire is probably due to the different chemical composition and treatment of the samples; however no data as to their purity were obtainable.

The average value of the temperature coefficient, .004455, for aluminium seems to be rather high compared to the value .00435 obtained by Dewar and Fleming for annealed aluminium. Since the purer the metal the greater the temperature coefficient, this would indicate that the sample tested was fairly pure, probably about 99.75% or 99.8% pure. It is also the case that the purer the metal the less the resistivity. It has been determined that the volume resistivity at 0° C. in micrehms per cubic centimeter for annealed aluminium, 99.66% pure is 2.4322. Since the value obtained here, i.e. 2.421, is lower, this would seem to verify the above estimate as to the ~~the~~ purity of the sample of aluminium tested.

Scheme of Connections of Thompson Double Bridge



Scheme of Connections for Test



Sample I Aluminium wire.

#10 B.W.C.

Temperature C°	Resistance of 50 cm	Temperature C°	Resistance of 50 cm
0.0	.001521	51.51	.001627
5.4	.0015455	56.15	.001650
10.5	.0015555	62.22	.0016865
17.83	.0014955	66.19	.001705
20.95	.001445	70.82	.0017375
26.21	.0014655	76.45	.001766
30.85	.0015030	80.26	.0017905
40.77	.001560	86.00	.001825
46.26	.0015925	91.92	.001856
		96.64	.001883
		101.75	.0019255

Diameter .13808"

Sample 11 Aluminium Wire.

"10 B.W.G.

Temperature C°	Resistance of 50 cm
0.0	.0015135
5.6	.001540
10.5	.0015565
14.7	.0014965
20.55	.001455
25.71	.001437
30.22	.0014025
35.30	.0015515
41.71	.001537
45.44	.0015865
50.40	.0016165

Diameter .13808 inches.

Sample 1 Annealed Copper Wire.

#10 B. & S. Gauge.

Temperature C	Resistance of 50 cm	Temperature C	Resistance of 50 cm
0.0	.001587	50.14	.001640
17.06	.001636	54.64	.001637
19.	.001639	58.16	.001635
30.06	.001645	65.64	.001618
51.07	.001651	66.74	.001655
68.06	.001661	70.16	.001637
74.10	.001670	74.64	.001630
85.00	.001679	85.16	.001631
89.71	.001690	87.61	.001635
90.00	.001711	89.67	.001655
94.10	.001679	86.75	.001679
99.06	.001735	90.75	.001661
45.11	.001710	95.64	.001652
46.61	.001715	100.15	.001660

Diameter .10175 inches.

Sample 7 Annealed Copper Wire.

.14 B. & O. Gauge.

Temperature C°	Resistance of 50 cm.	Temperature C°	Resistance of 50 cm.
0.0	.005055	46.50	.004716
6.50	.004975	48.50	.004750
18.4	.0049475	60.35	.004950
19.74	.004950	64.70	.005015
50.075	.004940	70.75	.005110
63.04	.004955	75.50	.005205
70.125	.004916	80.60	.005290
74.41	.004914	81.50	.005460
80.00	.004955	100.65	.005620

Diameter .065495 inches

Sample 4 - Annealed Conductor Wire.

" R&O Group.

Temperature C ^o	Resistance of 50 cm	Temperature C ^o	Resistance of 50 cm
0.0	.001780	57.64	.001660
6.15	.001910	60.05	.001747
30.05	.004096	64.00	.004100
35.76	.004160	70.75	.004305
39.05	.004250	76.05	.004331
35.00	.004355	80.76	.005061
40.15	.004400	86.04	.005117
46.60	.004584	90.50	.005304
55.50	.004650	96.00	.005505
		100.00	.005704

Diameter .00547 inch dia

Sample 5 Annealed Copper Wire.

#14 B & S Gauge.

Temperature C°	Resistance of 50 cm	Temperature C°	Resistance of 50 cm
0.0	.004030	52.43	.005008
10.0	.0041055	60.80	.005026
18.60	.004170	65.05	.005106
19.91	.004166	70.05	.005041
20.85	.004170	75.00	.005084
26.00	.004157	80.64	.005115
30.70	.004140	85.10	.005108
37.14	.004137	91.60	.005100
40.37	.004140	96.07	.005070
45.16	.004105	100.01	.005040
50.75	.004065		

Diameter .06221 inches.

Sample 6 Annealed Copper wire.

"14" Gauge.

Temperature C°	Resistance of 50 cm	Temperature C°	Resistance of 50 cm
0.0	.004337	45.73	.004365
6.03	.004355	50.55	.004384
6.36	.004358	53.82	.004397
6.5	.004351	60.73	.005310
10.10	.004357	65.33	.005310
11.61	.004331	70.51	.005346
12.99	.004417	76.74	.005400
21.26	.004457	81.10	.0054355
25.74	.004514	85.95	.005560
50.15	.004534	90.66	.005643
57.74	.004550	100.57	.005710
40.56	.004590		

Diameter .06541 inches

Sample 2 Annealed Copper Wire.

#10 B & S Gauge.

Temperature C°	Resistance of 50 cm	Temperature C°	Resistance of 50 cm
0.0	.000075	56.61	.001811
10.47	.001014	61.74	.001850
17.55	.001046	65.35	.001844
23.65	.001070	71.13	.001866
27.73	.001091	76.64	.0018905
50.60	.001104	81.65	.001818
35.57	.001124	86.17	.001880
40.70	.001143	90.30	.001850
45.07	.001165	95.75	.001880
50.0	.001185	100.50	.001867

Diameter .10726 inches.

Sample 10 Annealed Copper wire.

"0 B & N Gauge.

Temperature C. ^o	Resistance of 50 cm	Temperature C. ^o	Resistance of 50 cm
0.0	.0013165	51.35	.0014315
4.65	.0013505	51.3	.001431
9.50	.001365	55.04	.001500
15.05	.001395	60.65	.0015315
17.65	.0014075	65.14	.0015605
19.15	.0014205	69.10	.0015805
20.16	.001430	73.10	.0016105
25.1	.001455	77.55	.001650
30.00	.001485	85.35	.0016605
35.05	.001495	90.50	.001685
40.54	.001497	95.55	.001715
47.00	.001464	100.05	.001750

Diameter .11710 inches.

Sample C American Steel & Wire Co's Extra B.B. Steel Wire,
 "C" B. C.

Temperature C°	Resistance of 50 cm.	Temperature C°	Resistance of 50 cm.
0.2	.003456	53.65	.004546
7.875	.0035675	60.52	.004632
15.55	.003671	65.21	.004755
22.65	.003802	71.05	.004847
27.56	.004008	76.61	.0049875
30.06	.004055	81.53	.005081
36.56	.004185	86.44	.005196
40.38	.004264	91.13	.005300
46.32	.004377	95.60	.005415
51.45	.004430	100.0	.005531

Diameter .1655 inches.

Sample 7 Rebling's B.P. Steel 100.

W. P. .G.

Temperature C°	Resistance of 50 cm	Temperature C°	Resistance of 50 cm
22.0	.004166	60.48	.005751
22.45	.004621	66.37	.005861
22.54	.004799	70.36	.005957
26.18	.005066	81.70	.006168
39.21	.005153	85.31	.006296
56.04	.005235	91.71	.006454
41.71	.005554	96.50	.006535
46.35	.005481	101.50	.006630
51.00	.005554	106.10	.006750
56.00	.005660		

Diameter .1655 inches.

Sample 8 Roebling's Extra B.B. Steel Wire.

#8 B.B.G.

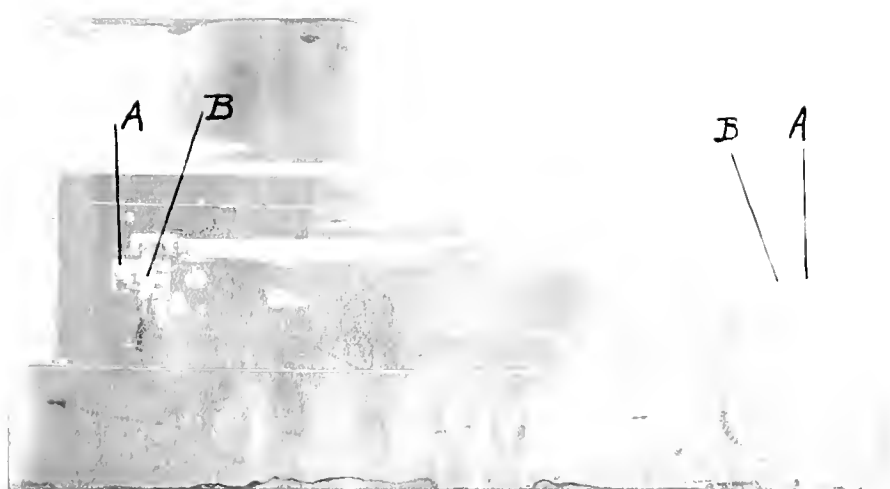
Temperature C°	Resistance of 50 c	Temperature C°	Resistance of 50 cm
0.0	.004624	60.25	.005854
12.55	.004927	66.55	.005977
19.50	.005008	70.90	.0060075
27.56	.0052015	76.22	.006130
50.57	.005358	81.01	.006210
56.32	.005387	86.40	.006401
41.32	.005471	91.51	.006527
46.71	.005581	101.05	.006758
50.80	.005652	96.51	.006646
55.85	.005756		

Diameter .1657 inches.

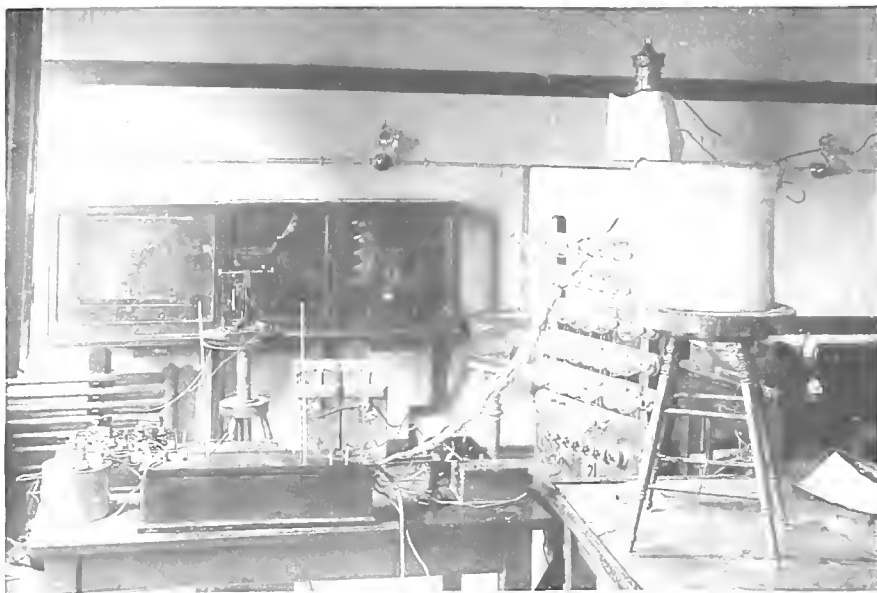
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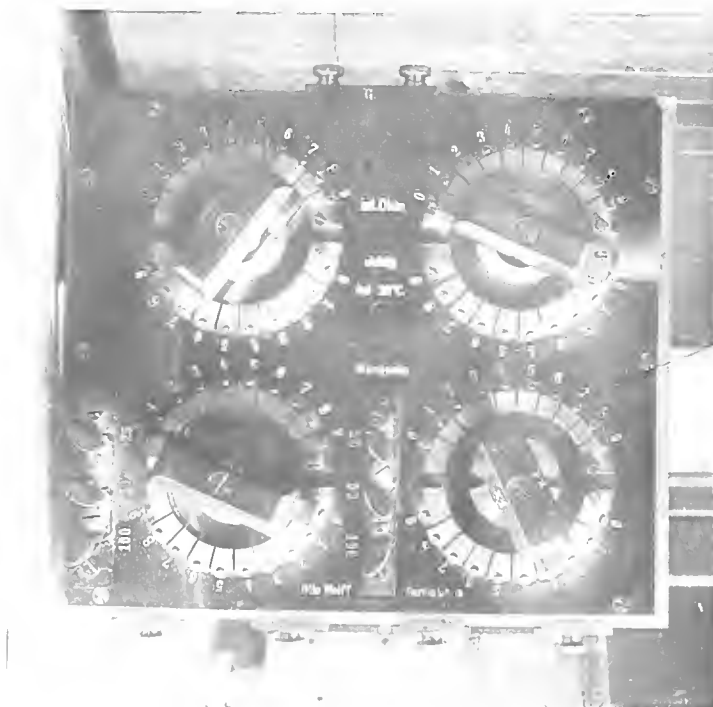
Heating Box.



*View of Cover of Heating Box showing
Knife Contacts.*



Arrangement of Apparatus.



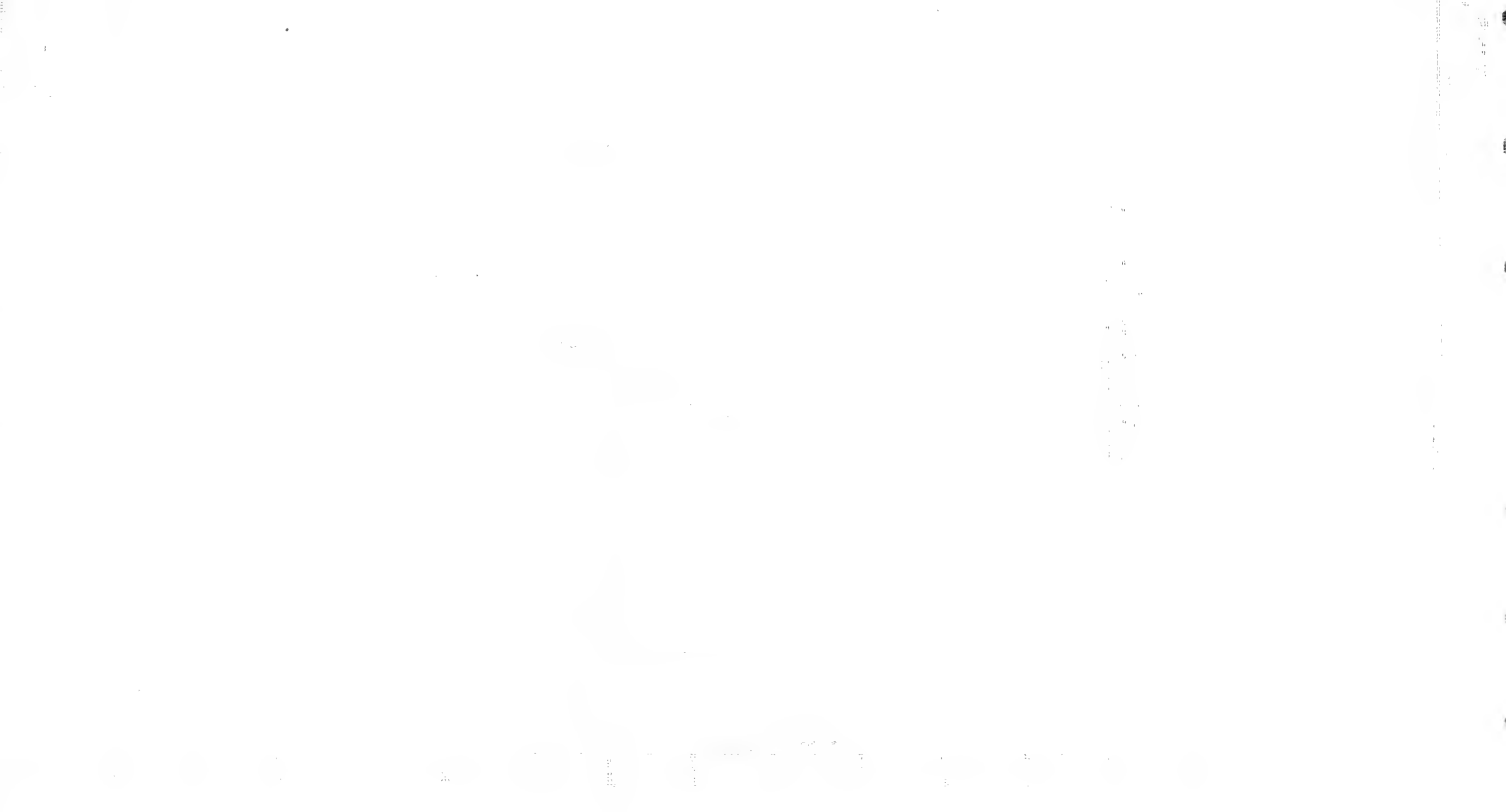
*View showing Dials and Ratio
Plugs of Thomson Double Bridge.*

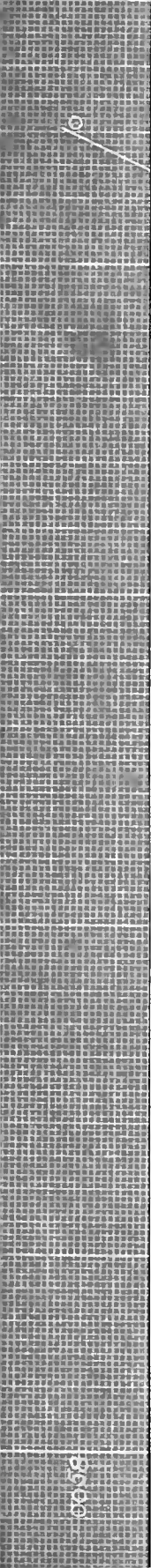


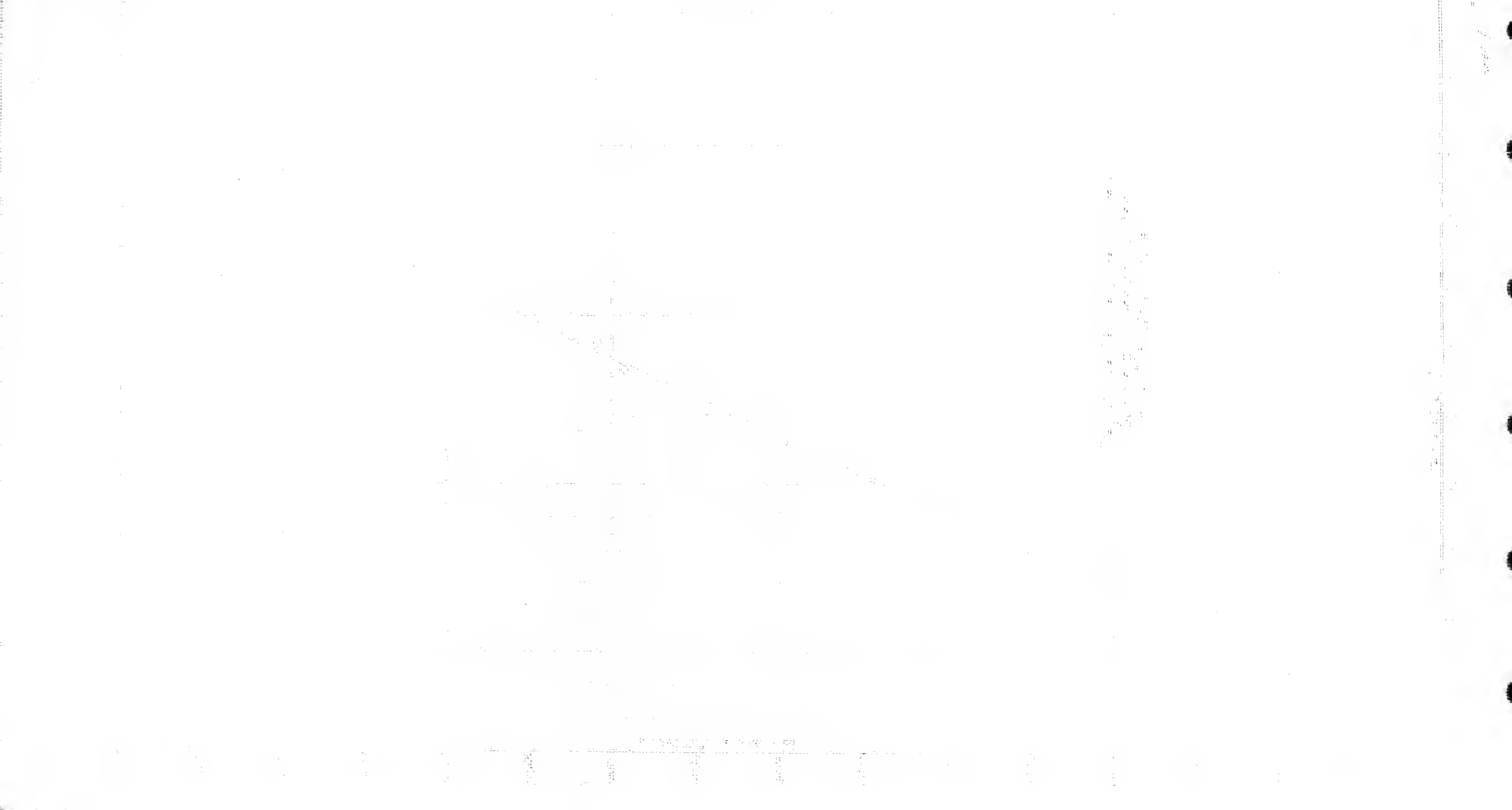
Arrangement of Apparatus.

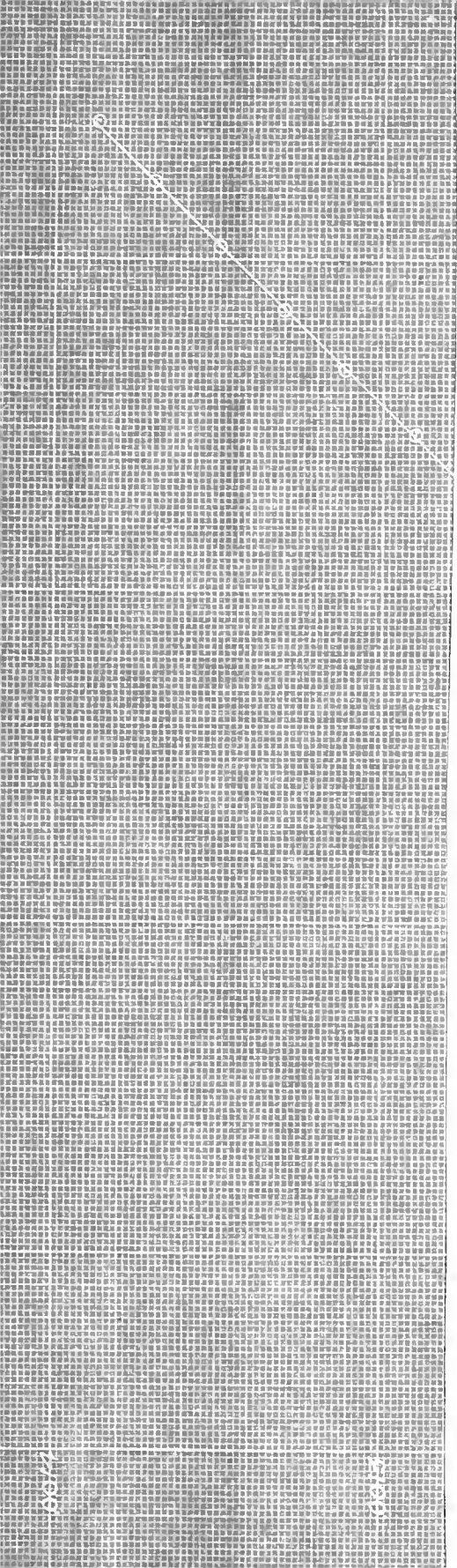
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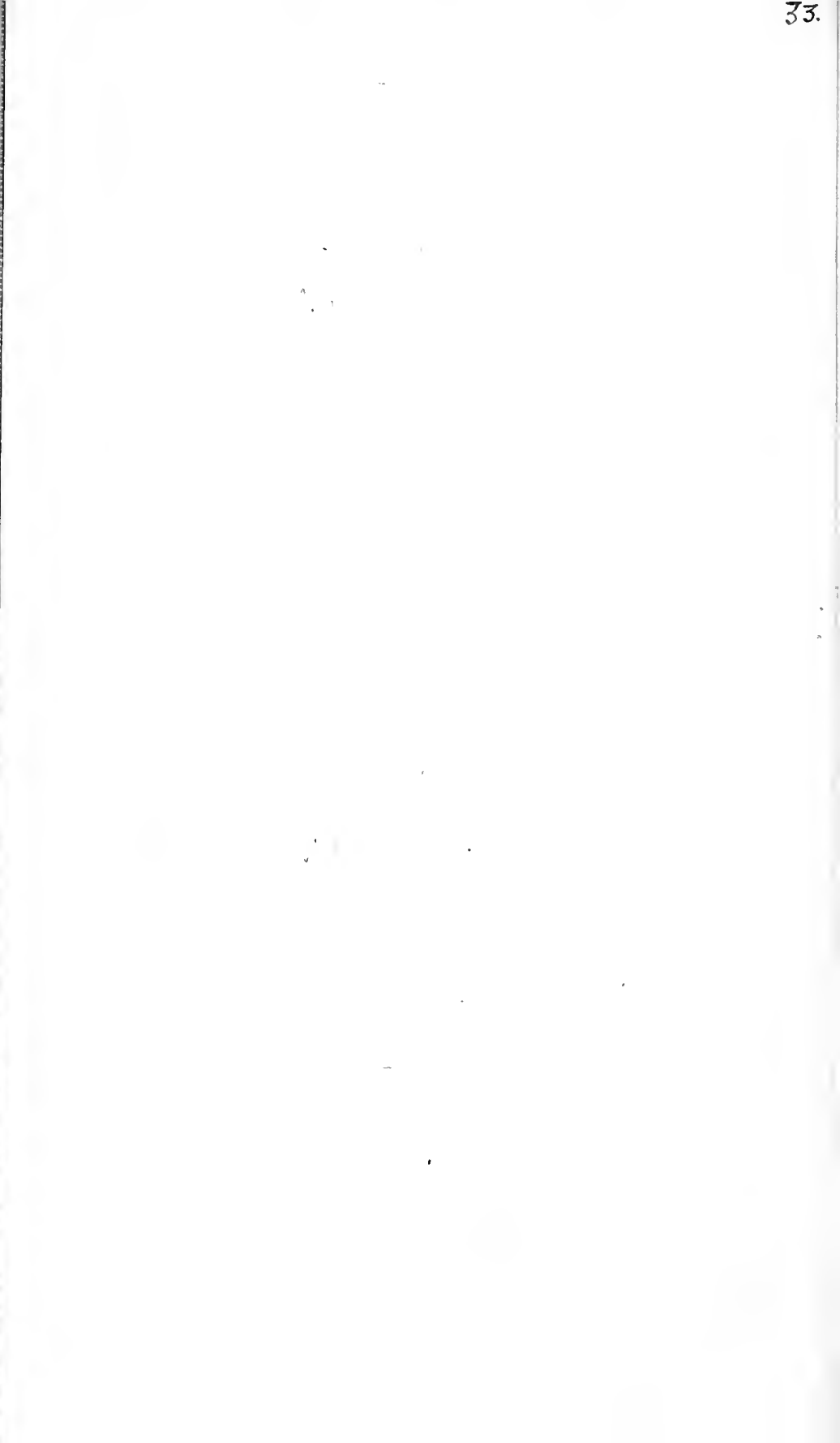


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1. The first part of the document is a list of the names of the persons who have been appointed to the various offices of the city government. The names are listed in alphabetical order, and each name is followed by the office to which the person has been appointed. The list is as follows:

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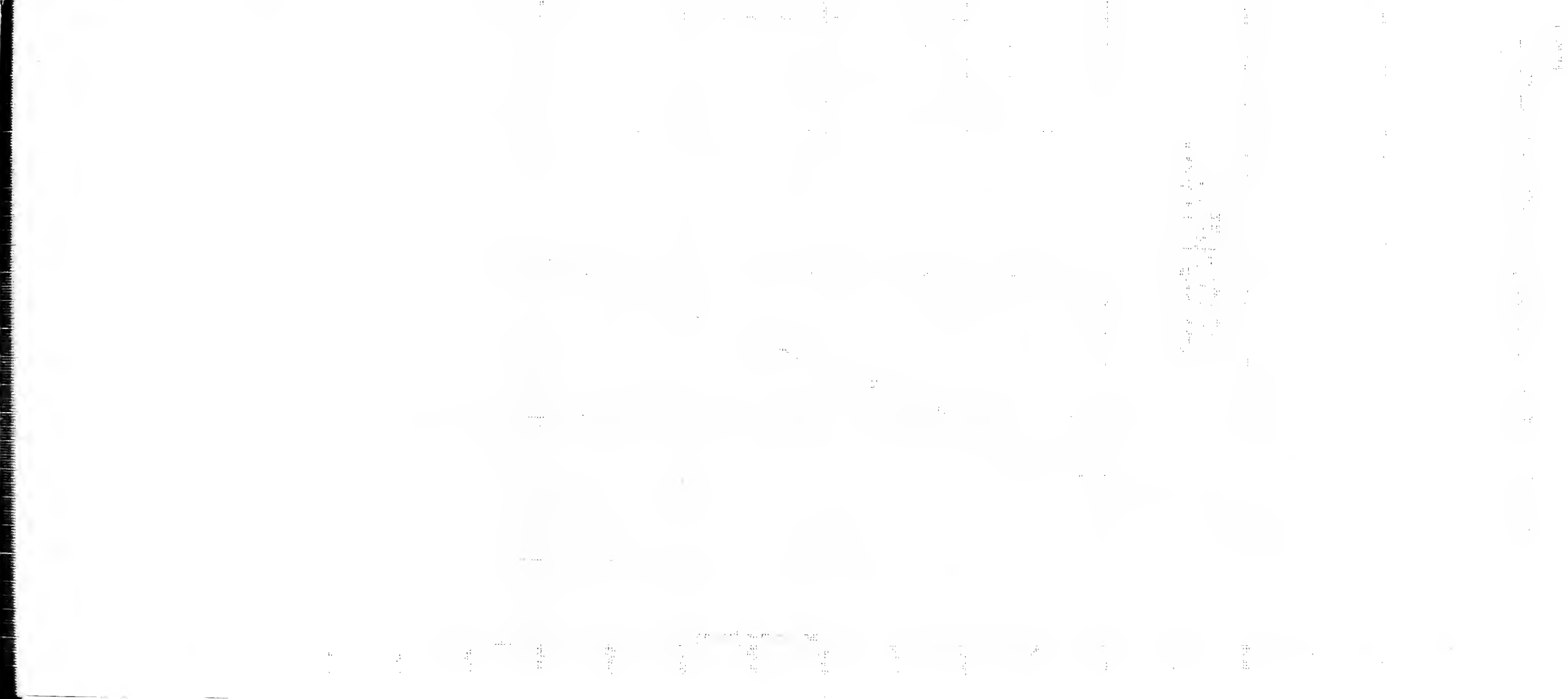
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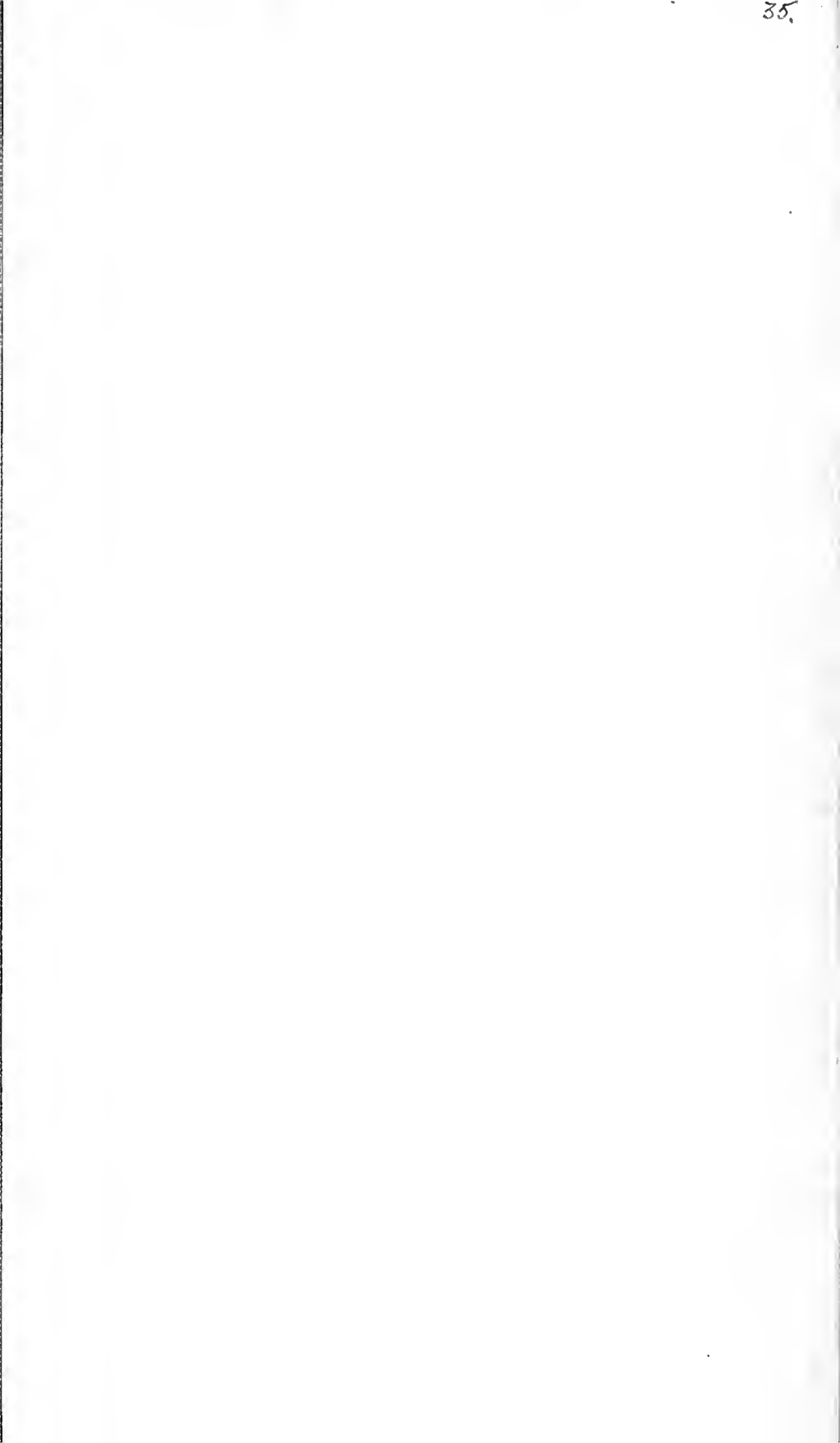
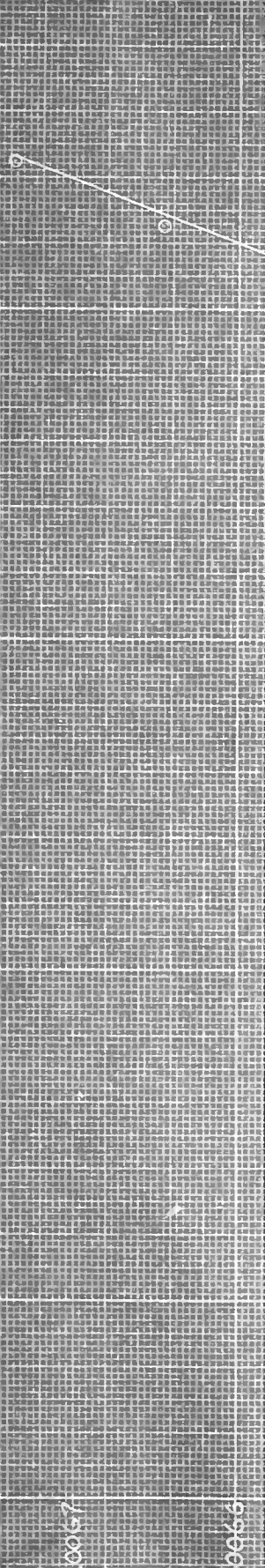
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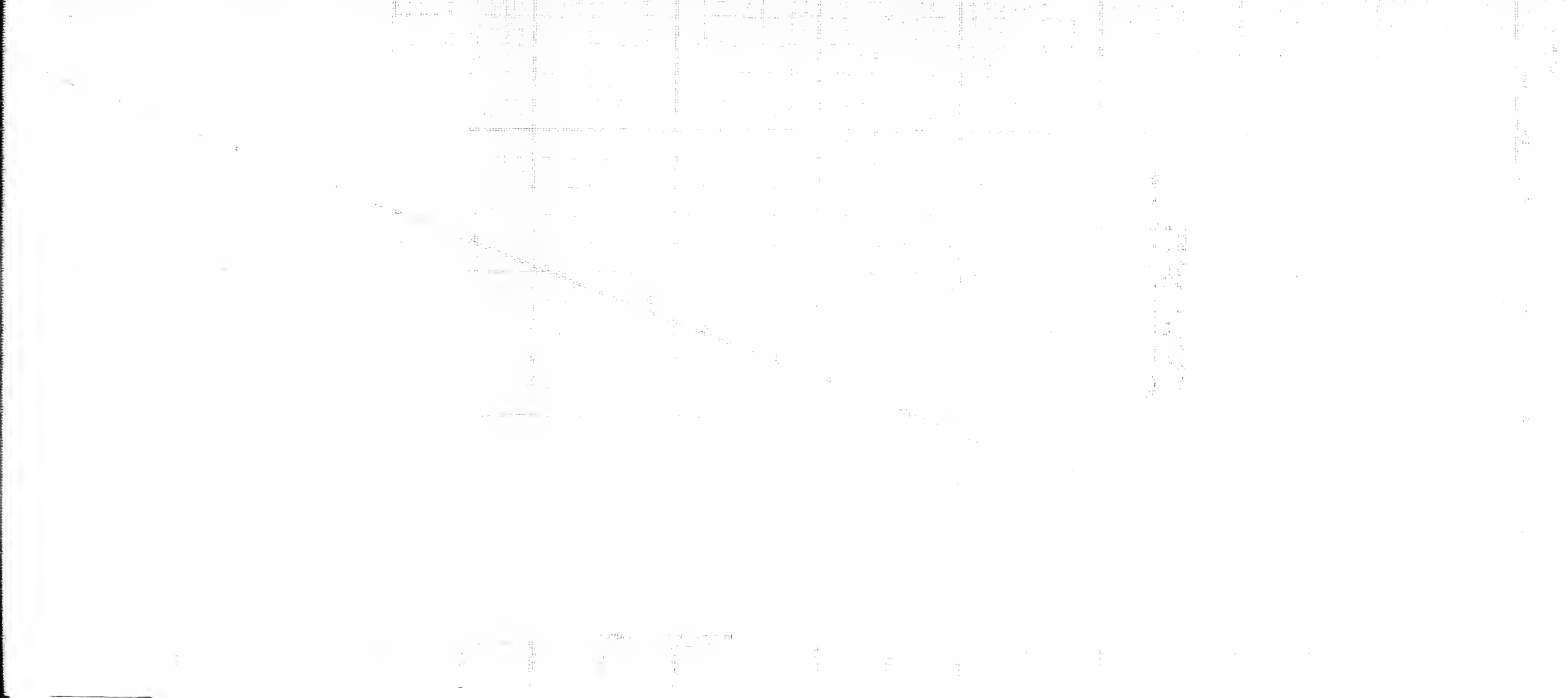
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1. The first part of the paper is devoted to a general discussion of the problem of the existence of solutions of the system of equations

2. The second part of the paper is devoted to a detailed analysis of the case of the existence of solutions of the system of equations

3. The third part of the paper is devoted to a detailed analysis of the case of the existence of solutions of the system of equations

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